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A state variable for characterizing water masses and their diffusive stability: spiciness

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Abstract

A state variable $\pi(\theta,s)$ most sensitive to isopycnal thermohaline variations and least correlated with the density field is proposed. π is constructed so that its diapycnal gradient $d_\rho\pi$ is related to the density gradient ratio R_ρ:$d_\rho\pi \approx (1+R_\rho)/(1-R_\rho)$. It is simultaneously a tracer conserved by isentropic motions, and an indicator of diffusive stability. It is useful for the combined description of interleaving and double-diffusive processes at the boundary between water masses. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Equation of state; Double diffusion; Tracer

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1. Introduction

The distributions of potential temperature θ and salinity s on surfaces of constant potential density $\rho(\theta,s,p = p_{\mathrm{ref}})$ are not independent: displaying both is redundant. Vertical profiles of either temperature or salinity also contains some information on the density stratification. It is therefore natural to introduce a state variable defined to be most sensitive to isopycnal thermohaline variations, and least correlated with the density field. The concept of such a variable is not new; it has been used, among many others, by Stommel (1962) to discuss the sharpness of the θ–s curve in deep water, by Munk (1981) to separate internal waves from intrusive fine-structure, by Olbers, Wenzel, and Willebrand (1985) as a tracer in an inverse model of the Atlantic circulation.

Flament (1986) defined and constructed such a state variable $\pi(\theta,s)$ in a way useful to characterize water masses, and also to indicate double-diffusive stability. Following Munk (1981), π was called 'spiciness', being largest for hot (spicy) and salty water. It has been widely used to study the California Current System (Niiler, Poulain, & Haury, 1989; Lynn & Simpson, 1990; Simpson & Lynn, 1990; Huyer, Barth, Kosro, Shearman, & Smith, 1998; Barth, Pierce, & Smith, 2000; Steger, Schwing, Collins, Rosenfeld, Garfield, &

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Gezgin, 2000). A number of papers in the present volume use this definition of spiciness, which this note makes readily accessible.

2. Definition

2.1. Spiciness as a state variable

A state variable is associated with a family of isopleths in the θ–s plane, intersecting the family of isopycnals. A water type can thus be defined interchangeably by its (θ,s) coordinates, or by its (θ,π) coordinates. The definition of spiciness should not depend on a specific choice of scales on the θ and s axes, and must, therefore, be invariant for all geometric transforms allowed in the θ–s plane. These consist of changes of scales along the axes; rotations are not allowed because θ and s have different physical dimensions. A scale-invariant constraint is to require that, at any point, the slopes of the isopycnals and of the spiciness isopleths are equal and of opposite sign (Fig. 1):

$$\frac{\partial_\theta \pi}{\partial_\theta \rho} + \frac{\partial_s \pi}{\partial_s \rho} = 0 \tag{1}$$

where $\partial_\theta \pi$ denotes the partial derivative $\partial \pi / \partial \theta$of π with respect to θ. This first order partial differential equation has a unique solution given an initial condition along a line distinct from its characteristics, themselves solutions of the associated ordinary differential equation:

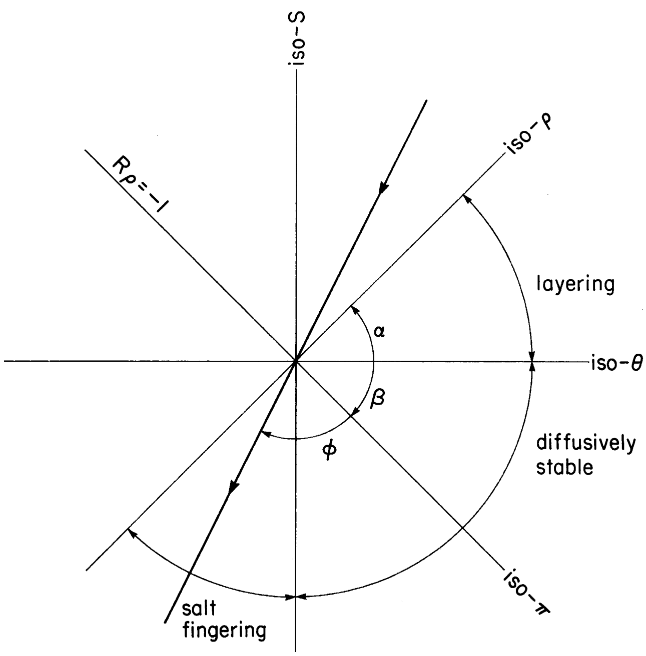

Fig. 1. A point of the θ–s plane with the scales chosen so that the isopycnal is at $\hat{\alpha} = 45°$ with the axes. The spiciness isopleth at that point is constructed so that $\hat{\alpha} = -\beta$; it is orthogonal to the isopycnal with this choice of scales. A vertical profile unstable to salt fingering is drawn, with $\varphi = -71.6°(R_\rho = 2)$; the arrow indicates the downward direction. The sectors corresponding to profiles diffusely stable, unstable to salt fingering and layering, and gravitationally unstable are also shown. Positive angles are counter-clockwise.

$$\partial_\theta \rho d\theta = \partial_s \rho ds \tag{2}$$

which defines a unique family of spiciness isopleths, regardless of the scales chosen for the axes.

Geometrically, at any point the isopycnals and the spiciness isopleths are symmetric with respect to the θ–s directions. When the scales are chosen so that the isopycnals are inclined at 45° with the axes at a point, the spiciness isopleths are orthogonal to the isopycnals at that point. Note that (2) is not a total differential, since in general the condition of integrability

$$\partial^2_{\theta s} \rho = 0 \tag{3}$$

is not satisfied, i.e. the isopycnals in the θ–s plane are curved.

The family of spiciness isopleths defined, it remains to label them: to assign a useful value of spiciness to each. Many choices are possible, for example one could select an initial condition $\pi_0(\lambda)$ along a line$\theta(\lambda)$, $s(\lambda)$, such as a θ–s curve, or a particular isopycnal. Instead of an exact initial condition along a particular line, an approximate 'initial' condition will be enforced in a least square sense over the domain of the θ–s plane containing most oceanic water masses. This condition will be selected to relate the diapycnal gradient of spiciness $d_\rho \pi$ to the diffusive stability of the water column.

2.2. Diffusive stability

The parameter controlling diffusive stability is the density gradient ratio (Turner, 1973):

$$R_\rho = \frac{\partial_\theta \rho d_p \theta}{\partial_s \rho d_s s} - \frac{\partial_\theta \pi d_p \theta}{\partial_s \pi d_p s} \tag{4}$$

where p is the hydrostatic pressure. Equation (4) provides a simple physical interpretation of (2): the spiciness isopleths are simply the lines along which the density ratio $R_\rho = -1$ (the adiabatic temperature gradient Γ has been neglected in (4); this is justified in the upper ocean and the main thermocline, where most water mass interleaving occurs).

The vertical gradients of ρ and π are:

$$d_p \rho = \partial_\theta \rho d_p \theta + \partial_s \rho d_p s = \partial_s \rho d_p s (1 - R_\rho) \tag{5a}$$

$$d_p \pi = \partial_\theta \pi d_p \theta + \partial_s \pi d_p s = \partial_s \pi d_p s (1 + R_e) \tag{5b}$$

so that

$$d_p \pi = \frac{\partial_s \pi}{\partial_s \rho} \frac{1 + R_\rho}{1 - R_\rho} d_p \rho = \mu \tan \phi d_p \rho \tag{6}$$

in which

$$\mu(\theta, s) = \tan^{-1} \frac{\partial_s \pi}{\partial_s \rho} \tag{7}$$

is an integrating factor of the ordinary differential Eq. (2), and in which the stability angle ϕ. has been defined such that:

$$\phi = \tan^{-1} \frac{1 + R_\rho}{1 - R_\rho} \tag{8}$$

The branch -90°<tan -1 <+90° is chosen for gravitationally stable stratifications, and the branch | tan -1 |>90° is chosen for gravitationally unstable stratifications.

The stability angle φ. is a good descriptor of diffusive stability. For gravitationally stable stratifications, diffusive layering occurs when $45° < \varphi < 90°$ and salt fingering when $-90° < \varphi < -45°$. For $|\varphi| < 45°$, the water column is diffusely stable. For $|\varphi| > 90°$, the water column is gravitationally unstable. The stability angle . is defined so that the stable range is centered on 0 (it differs by constant offsets from similar angles used by Ruddick (1983) and Washburn and Käse (1987). It has a geometric interpretation in the θ–s plane when the isopycnals are inclined at 45° with the s axis, as shown in Fig. 1.

If the 'initial' condition is chosen so that $\mu \approx 1$ over the entire oceanic range of θ–s, then π will have the same physical dimensions as ρ, and the stability angle will be approximately related to the diapycnal gradient of spiciness:

$$\tan\phi \approx \frac{\mathrm{d}\pi}{\mathrm{d}\rho}. \tag{9}$$

2.3. Constructing spiciness

Spiciness π is constructed using potential density referenced to the surface $p_{\mathrm{ref}} = 0$ following Fofonoff (1985). The solution is expressed as

$$\pi(\theta,s) = \sum_{i=0}^{5} \sum_{j=0}^{4} b_{ij}\theta^{i}(s-35)^{i} \tag{10}$$

and the b_{ij} are found by minimizing the weighted mean square error on (1) and on $\mu \approx 1$:

$$\varepsilon^{2} = \lambda_{1}\int\int \mathrm{d}\theta\mathrm{d}s\left[\frac{\partial_{\theta}\pi}{\partial_{\theta}\rho} + \frac{\partial_{s}\pi}{\partial_{s}\rho}\right]^{2} + \lambda_{2}\int\int \mathrm{d}\theta\mathrm{d}s\left[\frac{\partial_{s}\pi}{\partial_{s}\rho}-1\right]^{2} \tag{11}$$

The integration is carried over the domain delimited by (0.32), (30.35), (30.38) and (0.35), containing most of the water masses of the oceans (see ; and Levitus, 1982). The weights $\lambda_1 = 0.8$ and $\lambda_2 = 0.2$ were chosen to emphasize the geometric constraint between the spiciness isopleths and the isopycnals; the resulting coefficients are not very sensitive to this choice. The coefficients b_{ij} are given in Table 1; as a convention, $b_{00} = 0$ so that $\pi(0,35) = 0$. The families of isopycnals and spiciness isopleths are shown in Fig. 2.

The rms and maximum errors on (1) are 2×10^{-3} and 5×10^{-3}, and the rms and maximum errors on $\mu \approx 1$ are 1.4×10^{-2} and 4×10^{-2} respectively, the maximum values being confined to the edges of the integration domain. For the critical cases of the stability limits $\varphi = \pm 45°$, the corresponding rms and maximum errors on φ are 0.4° and 1.2° are negligible for practical purposes.

3. Example

Fig. 3 shows a profile through a Mediterranean water eddy ('salt lens'), south of the Azores (Armi & Zenk, 1984). The corresponding θ–s curve is shown in Fig. 4. Fig. 5a shows the profile of spiciness using potential density as the ordinate. The isotherms and isohalines are overlaid. In this representation, they are approximately at ±45° with the ρ–π axes, and indicate the limits of diffusive stability: a segment of the profile steeper than an isotherm or an isohaline is diffusely stable. Fig. 5b shows the corresponding stability angle profile, computed over vertical intervals larger than the scale of the fine-structure observed below the salt lens.

The main thermocline, from A to B, is slightly diffusively unstable to salt-fingering, with$\varphi \sim -72°(R_\rho \sim 2)$. A small diffusively-stable section BC is followed by the top of the salt lens CD, unstable to layering, with $\varphi \sim -72°(R_\rho \sim 0.5)$. The core of the lens DE is diffusively stable. Below the lens, there is

Table 1
Coefficients of the polynomial expansion (10). In the last column, (k) means •10^k. (available at http://satftp.soest.hawaii.edu/spice/)

i	j	b_{ij}	k
0	0	0.0	
0	1	7.7442	(−1)
0	2	−5.85	(−3)
0	3	−9.84	(−4)
0	4	−2.06	(−4)
1	0	5.1655	(−2)
1	1	2.034	(−3)
1	2	−2.742	(−4)
1	3	−8.5	(−6)
1	4	1.36	(−5)
2	0	6.64783	(−3)
2	1	−2.4681	(−4)
2	2	−1.428	(−5)
2	3	3.337	(−5)
2	4	7.894	(−6)
3	0	−5.4023	(−5)
3	1	7.326	(−6)
3	2	7.0036	(−6)
3	3	−3.0412	(−6)
3	4	−1.0853	(−6)
4	0	3.949	(−7)
4	1	−3.029	(−8)
4	2	−3.8209	(−7)
4	3	1.0012	(−7)
4	4	4.7133	(−8)
5	0	−6.36	(−10)
5	1	−1.309	(−9)
5	2	6.048	(−9)
5	3	−1.1409	(−9)
5	4	−6.676	(−10)

a region strongly unstable to salt-fingering, with $\varphi \sim -84°(R_\rho \sim 1.25)$, associated with conspicuous thermohaline inversions and staircase-like structures.

In Fig. 5a, the stability angle is simply the angle between the tangent to the curve and the vertical (cf. Fig. 1 rotated 45° clockwise). This representation has thus the advantage of displaying the stability regime graphically, without the need to explicitly compute scale-dependent derivatives as in Fig. 5b.

4. Discussion

Analogous variables were proposed by Veronis (1972), and Jackett and McDougall (1985). Veronis (1972) constrained the isopleths of his variable τ_V (θ,s) to be orthogonal to the isopycnals in the θ–s plane. However his definition depends on the scaling of the axes: geometric orthogonality is lost when the scales are altered, i.e. his constraint is invariant for rotations, but not for scale changes, which should be required for a state variable.

The variable τ_{JMD} introduced by Jackett and McDougall (1985) differs from π in three important aspects. Firstly, they enforce the constraint (1) exactly on a single spiciness isopleth, instead of in a least square

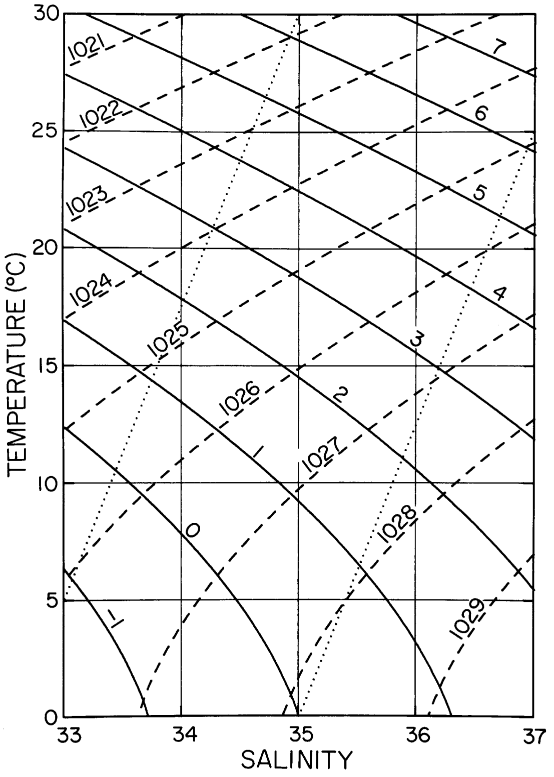

Fig. 2. The families of isopycnals (dashed) and spiciness isopleths (solid) with the reference pressure at the surface. With the scales chosen for this graph, the lines are orthogonal at about 8.5 °C. The domain over which the error (11) was minimized is delimited by dotted lines.

sense over the domain of ocean water masses. Secondly, they require that, along all isopycnals, $d\tau_{\mathrm{JMD}} = 2\rho^{-1}\partial_s\rho ds$; although this is also related to double-diffusive convection (i.e. Ruddick & Turner, 1979), Eq. (9) provides a more transparent interpretation in terms of diffusive stability, with direct applications (Ruddick, 1983). Thirdly, τ_{JMD} is dimensionless, whereas π has the same physical dimensions as density, yielding simple and elegant expressions for (6), (7) and (9). For practical purposes and over the range of oceanic water masses, linear conversions between τ_{JMD} and π can be obtained by:

$$\tau_{\mathrm{JMD}} = 0.97\pi + 23.8 \tag{12a}$$

and

$$\pi = 1.03\tau_{\mathrm{JMD}} - 24.5 \tag{12b}$$

with standard errors less than 0.025 in units of either π or τ_{JMD}.

5. Conclusions

Spiciness, as defined here, is ideally suited for the combined description of interleaving and double-diffusive processes at the boundary between different water masses, such as, for example, shelf-slope and

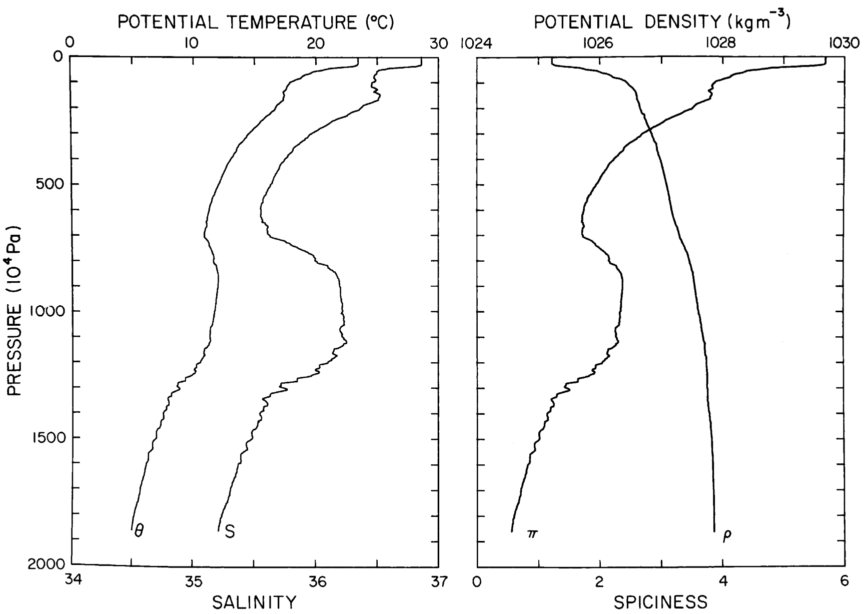

Fig. 3. Vertical profiles through a Mediterranean salt lens, using pressure as the ordinate, taken on 17 September 1984 at 31°42′N; 22°00′W (courtesy of L. Armi). (a) salinity and potential temperature (referenced to the surface), (b) potential density and spiciness.

coastal upwelling fronts, or marginal seas outflows. It has the practical advantage of lumping together the concept of a state variable decoupled from density, and the concept of an indicator of diffusive stability.

The ρ–π plane has an elegant geometrical property: isotherms and isohalines are approximately straight lines, the non-linearity of the equation of state being nearly entirely contained in their variable spacing. In other words, with properly engraved non-linear rulers for θ and for s, a θ–s diagram could be constructed entirely by hand, without recourse to a calculator or to special pre- printed isopycnal paper.

A word of caution: spiciness, just like potential density, is only useful over limited vertical excursions near the pressure to which they are referenced; for large vertical ranges, the slope of the isopycnals and spiciness isopleths vary significantly with pressure, and generalization of the polynomial expansion to include a reference pressure dependence is needed. The maximum pressure-induced slope error for spiciness between the surface and 1000 m depth, amounts to 1.5° at $\theta = 20$ and $s = 35$, and 3° at $\theta = 10$ and $s = 35$.

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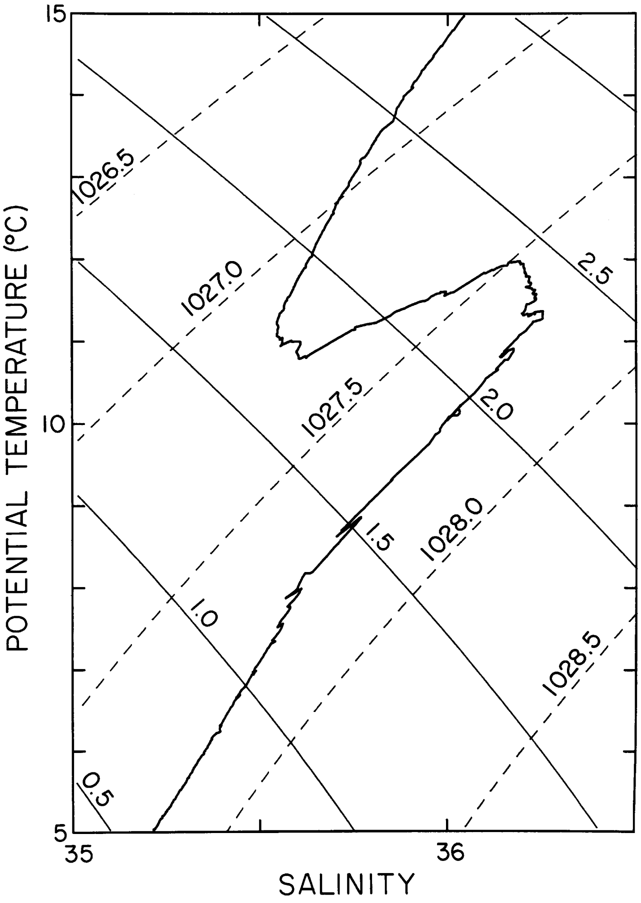

Fig. 4. θ–*s* curve for the profile shown in Fig. 3. The isopycnals and isospices are overlaid.

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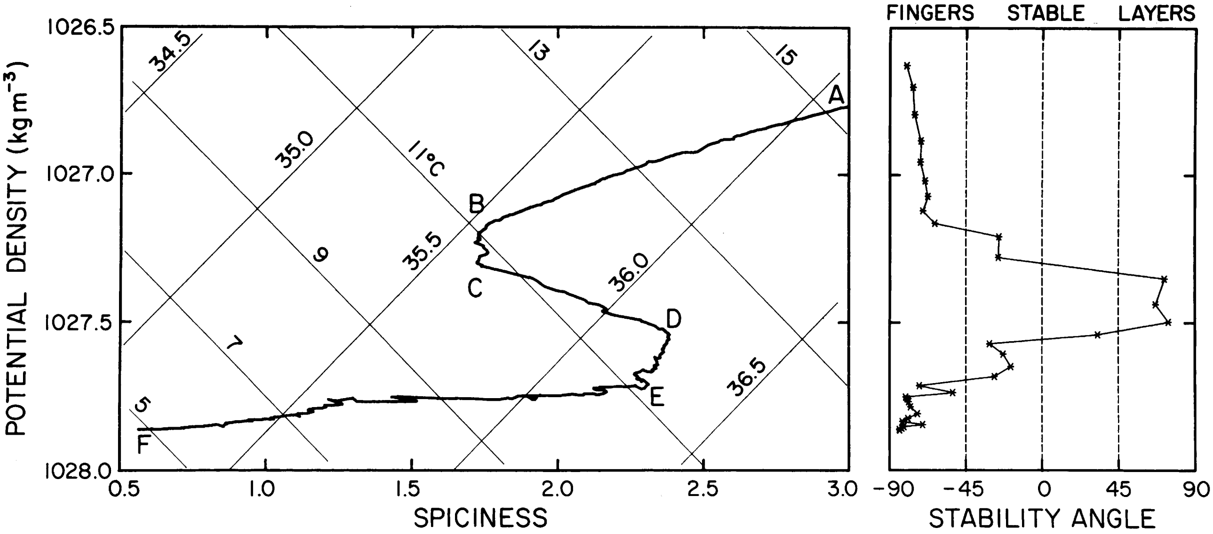

Fig. 5. (a) profile of spiciness using potential density as the ordinate; the profile is diffusely stable where the slope is steeper than the isotherms and isohalines, indicated by dashed lines. (b) corresponding profile for the stability angle φ, computed by a least-square fit over 50 m intervals.

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